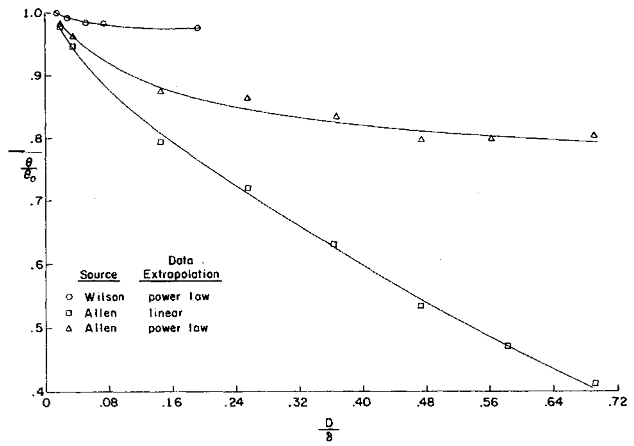

Fig. 5 Effect of probe size on experimental momentum thicknesses.

caused by probe-tip geometry, as can be seen in Fig. 4 in which the data of Fig. 3 have been replotted to isolate the effects of probe tip.

In an attempt to discover what were the true causes of the divergent results, the present author performed three additional checks: 1) the probe of configuration A was made structurally rigid and tested to see if it had been deflecting under airload; 2) configuration F was run at several values of α to see if the small angle-of-attack difference between the probes of Refs. 2 and 3 caused the interference results to differ; and 3) configurations A and C were tested in another wind tunnel to see if the present results were caused by some anomaly in the original test setup. Neither of these three further checks made any significant difference in the measured results.

To illustrate the difference in the results obtained in these two experiments, Fig. 4 of Wilson's Note[1] showed the effects of probe size on the momentum thickness calculated from the measured profiles of the two experiments. The values of the momentum thickness for large probe data, however, depend on the type of data extrapolation used between the wall and $D/2$. The momentum thickness comparison in Wilson's Note was made between data in which these extrapolations were not the same: Wilson and Young used a power-law extrapolation whereas Allen, who was attempting to show the errors that would occur if large probes were used in conjunction with standard data integration procedures, used a linear extrapolation.

In order to make the momentum thickness comparisons based on a similar data integration procedure, Allen's momentum thicknesses have been recalculated assuming a power-law extrapolation, and the results are shown in Fig. 5. The recalculated momentum thicknesses show much less effect of probe size. However, the effects are still larger than those shown in Wilson and Young's data, as would be expected from the fact that Allen's velocity profiles contained larger probe interference effects than those of Wilson and Young.

In summary, the author finds it frustrating not to be able to establish the reason for the differences between the pitot-probe interference results of Refs. 2 and 3; however, he feels that the additional data reported herein have shown that the differences are not caused by differences in the probe-tip and support geometrics as stated by Wilson.[1]

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Numerical Procedure for Analyzing Langmuir Probe Data

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Nomenclature

A	= probe collection area, m^2
B_1	= primary electron current intercept, amp
B_2	= primary electron current slope, av^{-1}
B_3	= $I_{sat}\exp(-\phi_p/T_m)$, amp
B_4	= $1/T_m$, ev^{-1}
B_i^0	= initial estimates of the parameters $B_i (i=1,4)$
i_p	= primary electron current at plasma potential, amp
I	= probe electron current, amp
I_{sat}	= Maxwellian electron saturation current, amp
m_e	= electron mass, kg
n_m	= Maxwellian electron number density, m^{-3}
n_p	= primary electron number density, m^{-3}
q	= electronic charge, C
T_m	= Maxwellian electron temperature, ev
V	= probe potential, volt
δB_i	= corrections to the estimates of the parameters $B_i (i=1,4)$
ζ_p	= primary electron energy, ev
ϕ_p	= plasma potential, volt

Introduction

THE analysis of Langmuir probe data obtained in electron bombardment ion thrusters that use mercury as the propellant is complicated by the presence of two distinct groups of plasma electrons. One group has a Maxwellian energy distribution function with a temperature of approximately 5 ev, and the other group has an isotropic monoenergetic distribution function with an energy of approximately 30 ev. This latter group is sometimes referred to as a "primary" electron group. Although the two-group theory is only an approximation, it does have a physical basis and has been verified by experiment.[1,2]

If the plasma contains only Maxwellian electrons the analysis of Langmuir probe data is straightforward since the current to an infinite planar probe varies exponentially with probe voltage in the retarding field region (probe voltage less than plasma potential), and remains constant in the accelerating field region (probe voltage greater than plasma potential). Experimentally there is a slight variation in probe current with probe voltage beyond the plasma potential and this phenomenon is attributed to an increase in the dimensions of the plasma sheath surrounding the probe. The graphical procedure used in analyzing probe data when only Maxwellian electrons are present in the plasma consists of plotting the logarithm of the probe current as a function of probe voltage. This results in a curve which can be approximated by two straight line segments which intersect at the plasma potential. The slope of the straight line approximation in the retarding field region determines the electron temperature and the

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current at the intersection of the two straight lines gives the electron number density.

If the plasma contains both primary and Maxwellian electrons, the logarithm of the probe current is no longer a linear function of the probe voltage and the analysis becomes much more difficult. A graphical method[1] for analyzing this type of trace has been devised and is based on the existence of a linear segment of the current-voltage curve in the retarding field region of the trace. However, the current-voltage curves are in some cases far from linear in the retarding field region probably due, at least in part, to a spread in the primary electron energy distribution function. This nonlinearity is the major source of error when applying the graphical analysis technique. Figure 1 is a plot of electron current vs probe voltage for a two-group plasma having properties typical of an ion thruster. The curve was generated using analytical expressions for the variation of primary and Maxwellian electron current as a function of probe potential and is presented to illustrate the absence of a well defined linear portion of the trace even when the two-group assumption is satisfied. Actual probe data exhibit a smooth region near the plasma potential (shown as a broken line in Fig. 1) which is attributed to the existence of electrical noise in the discharge chamber. This paper describes a numerical procedure which has been developed for analyzing Langmuir probe data assuming a two-group plasma.

Theory

The assumption that the plasma contains two distinct groups of electrons enables one to express the electron current to a Langmuir probe as the sum of two currents. The current due to primary electrons varies linearly with probe voltage for an infinite planar probe. The other current, due to Maxwellian plasma electrons, varies exponentially with probe voltage in the retarding field region. Hence, the current to the probe in the retarding field region of the trace can be written as

$$I=B_1+B_2V+B_3 \exp (B_4V) \tag{1}$$

Given a set of data pairs (I, V) which have been measured using a Langmuir probe, the plasma properties can be calculated in a straightforward manner if the unknown coefficients $B_i(i=1,4)$ can be determined by a curve-fitting technique. Note, however, that the functional relationship for the current is nonlinear in the parameters B_i. In this type of problem, the standard linear least-squares method is not applicable and one must apply a nonlinear method.

Procedure

The procedure used to determine the unknown coefficients B_i is known as a least-squares differential-correction technique.[3] The method requires the user to provide reasonable initial estimates of the coefficients B_i. These estimates result in a set of linear algebraic equations the solution of which yields the first-order corrections δB_i to the initial estimates of the parameters B_i. If the absolute value of any of these corrections is larger than some specified value, the initial estimates B_i^0 are replaced by the quantity $B_i^0+\delta B_i$ and the procedure repeated.

As one might expect, convergence of the differential-correction technique depends on how closely the initial estimates B_i^0 approximate the actual values of the parameters B_i. Note, however, that for this particular problem if the parameter B_4 is known then the function becomes linear in the remaining unknown parameters $B_i(i=1,3)$ which can be determined by the method of linear least squares. This is important, since one can provide a very reasonable estimate of the unknown parameter B_4 (the reciprocal of the Maxwellian electron temperature which for a mercury plasma would be expected to be around 5 ev). With this initial estimate, the method of linear least squares can be used to find the remaining initial estimates of the parameters $B_i(i=1,3)$ for the differential-correction technique.

The procedure described uses only the data in the retarding field region of the probe trace. Since the primary electron current must be positive, the minimum probe voltage for which Eq. (1) is valid is given by

$$V=-B_1/B_2 \tag{2}$$

For each probe voltage which is greater than this value, the Maxwellian current is found by subtracting the quantity B_1+B_2V from the total current. The logarithm of this current (Maxwellian electron current) is a linear function of probe voltage, and a linear least-squares fit of these data in both regions of the probe trace yields the equations of the straight line segments. Equating these expressions yields the plasma potential and electron saturation current. Finally, the number densities and primary electron energy are calculated from the expressions

$$n_m=[2\pi m_e/qT_m]^{1/2}\, I_{sat}/Aq \tag{3}$$

$$n_p=[8m_e/q\zeta_p]^{1/2}\, i_p/Aq \tag{4}$$

$$\zeta_p=\phi_p+B_1/B_2 \tag{5}$$

Since the slope of the logarithm of the Maxwellian current in the retarding field region of the probe trace is the reciprocal of the electron temperature, this value can be compared to the parameter B_4 calculated by the differential-correction technique. These values should agree and the comparison can be used as an indication of the accuracy of the method.

With all the plasma properties determined, the probe current can be calculated as a function of probe voltage by use of Eq. (1). These data can be compared to the measured values and an error computed which can then be used as an indication of how accurately the plasma properties have been determined and also how well the two-group assumption is satisfied.

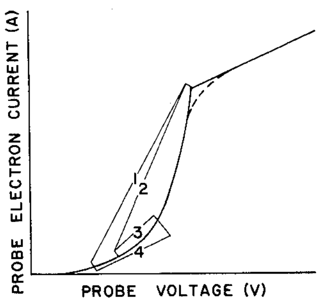

Fig. 1 Analytically generated Langmuir probe trace assuming a two-group plasma.

Results

The procedure described above was programed in the Fortran IV language. Three separate studies were conducted to a) verify the method, b) estimate the sensitivity of the results to random errors due to instrumentation, analog to digital conversion, etc., and c) compare the plasma properties determined by the numerical procedure to those determined graphically for input data obtained in an operating ion thruster.

The program was verified by using analytic input data which were generated by use of Eq. (1) and assumed values of the parameters B_i. This investigation revealed that when the iteration technique converged, the results were exact (to four significant figures), unique, and independent of the initial estimate of electron temperature. Convergence was fastest when the initial estimate of the temperature was high rather than low. However, an estimate which was greater than twice the actual value did not always result in convergence. The results were also found to be independent of the region of the probe curve used as input. That is, input data taken at one volt increments from each of the regions indicated in Fig. 1 gave exact results. Over a reasonable voltage range the results were found to be independent of the number of data pairs input. Results for input data taken from regions 1 and 2 in Fig. 1 were exact when the voltage increment was increased to five volts which reduced the number of input data pairs from 26 to 6.

The sensitivity of the results to random error was investigated by superimposing a random error on the analytically generated input data. This investigation indicated that the results were sensitive to the region of the curve used as input as well as the voltage increment. Plasma property errors were found to be minimal when the input data were taken over a 20-25 v range (region 1 or 2) at a 1-volt increment.

Several Langmuir probe traces obtained from an operating ion thruster were analyzed by the graphical procedure[1] and the numerical procedure outlined above. In the analysis, the ion current to the probe was considered negligible and the probe current was assumed to be electron current only. Comparison of the plasma properties determined by these methods indicated good agreement for the plasma potential and Maxwellian number density, while poor agreement was observed for the Maxwellian electron temperature and primary electron energy and density. The average absolute difference, expressed as a percentage of the numerically determined plasma property, was 3%, 14%, 36%, 37%, and 285%, respectively. The numerical procedure determined electron temperatures and energies which are more consistent with expected values than those determined graphically. For example, on the thruster centerline the numerical procedure resulted in an average Maxwellian electron temperature and primary electron energy of 4.1 ev and 32 ev, respectively. The corresponding graphically determined averages were 2.4 ev and 16.2 ev. The Maxwellian electron temperature expected to exist in mercury bombardment thrusters is about 4-5 ev, while the primary electrons would be expected to have an energy in the 30-35 ev range when operating at a 37 volt anode potential.

Conclusions

Based on these studies, the following conclusions regarding the applicability of the numerical procedure are drawn: 1) For a given set of input data the results of the analysis are unique and independent of the initial estimate of electron temperature. 2) Convergence is fastest when the electron temperature is over-estimated. However, an estimate which is greater than twice the actual temperature may not result in convergence. 3) The results are sensitive to the region of the curve used as input and also to the voltage increment. Thus, in order to minimize the effect of random error the input data should be taken over a wide voltage range ($\sim 25V$) with a small enough voltage increment ($\sim 1V$) to give a fairly large number of input data pairs. 4) Plasma properties determined by the numerical procedure have either shown good agreement with those determined graphically or, where agreement was poor, have been closer to expected values.

With a suitable data acquisition system, the program could be used to provide real time plasma diagnostic information for an operating ion thruster. Although the procedure has been developed for thin plasma sheath analysis, it can easily be extended to thick sheath analysis since the nonlinear curve fitting technique is applied to the retarding field region of the probe trace which is independent of sheath dimensions.

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Turbulent Boundary Layers with Wall Injection

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RECENTLY, considerable attention has been given, from both the analytical and experimental points of view, to the development of a turbulent boundary layer with continuous injection or suction at the surface.[1-3] The more challenging situation of a turbulent boundary subjected to a step change in surface injection has been studied experimentally[4-7] and represents a rather severe test of the present calculation methods.

In this Note, the elegant method of Bradshaw, Ferriss, and Atwell[8] is used to calculate the perturbed flow region downstream of a discontinuity in surface injection. Although this method is not the most sophisticated now available, the solution of the differential equations for mean velocity and Reynolds shear stress (or, more accurately, turbulent kinetic energy) by the method of characteristics is well suited to the study of the propagation of the disturbance downstream of the surface change. Essentially, the turbulent energy equation is converted into an equation for shear stress by assuming three empirical functions relating the turbulent intensity, diffusion and dissipation to the shear stress profile. The relevance of these assumptions to the boundary layer with surface injection or suction is discussed in detail elsewhere.[9] Here it will suffice to state that κ, the von Karman constant, and A, the additive constant, in the logarithmic velocity profile

$$U/U_\tau = \kappa^{-1}\ln(yU_\tau/\nu + A$$

where U_τ is the friction velocity $= \tau_w^{1/2}$, τ_w is the kinematic wall shear stress, ν is the kinematic viscosity), were taken to be 0.40 and 2.0, respectively; i.e., the values generally accepted for smooth wall boundary layers with zero mass transfer and pressure gradient.

Before comparing the results of the method for the step change in surface injection, a comparison was made with the experimental data of Nayak[10] and of Wooldridge and Muzzy[11] for the turbulent boundary layers with uniform surface injection and zero pressure gradient. Satisfactory agreement between the calculated and experimental mean velocity profiles and Reynolds shear stress profiles was obtained. For the case of discontinuous surface injection, the data of Simpson[7] showed that the value of A could be used unchanged. Only small discontinuities were considered, in order not to invalidate the assumptions inherent in the calculation method of Bradshaw et al.[8] Comparisons were made with the experimental data of Levitch[6] and Simpson.[7] In the case of Levitch, the wall injection velocity V_w was decreased abruptly from 0.0045 U_1 (U_1 is the freestream velocity), upstream of the step to zero downstream. Good agreement was obtained for the mean velocity and shear stress

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